LSND RESULTS AND THEIR IMPLICATIONS

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The combined 1993, 1994, and 1995 data from the LSND experiment shows a statistically compelling excess of events of the type expected for $\bar{\nu}_\mu \to \bar{\nu}_e$ neutrino oscillations. An electron between 36 and 60 MeV is identified by Cherenkov and scintillation light from $\bar{\nu}_e p \to e^+ n$, and if a γ is tightly constrained to be correlated with it from $np \to d\gamma$ (2.2 MeV), then 22 such events are observed, but only 4.6 ± 0.6 are expected from backgrounds. The probability that this is a fluctuation is $< 10^{-7}$. If subsequent analysis shows a similar effect from the independent channel $\nu_\mu \to \nu_e$, then this would imply a neutrino mass difference which would contribute significantly to the dark matter of the universe. Explaining also the solar and atmospheric neutrino deficits results in a neutrino mass pattern which gives a cold + hot dark matter mix which fits the structure of the universe on all scales and requires a critical density universe and a universe age compatible with that of the oldest stars. This mass pattern involves a sterile neutrino, evidence for which may come from the need for it in producing heavy elements by supernovae and for blowing off the supernova mantle.

1 LSND Experiment

The existence of neutrino mass would have profound effects on particle physics, since that is outside the Standard Model, and on cosmology, since the $\sim 10^2$ relic neutrinos/cm^3 of each type would contribute significantly to the mass of the universe and hence to the formation of its structure. Evidence for neutrino mass comes from the LSND (Large Scintillator Neutrino Detector) experiment, which uses 800-MeV protons from the Los Alamos Meson Physics Facility accelerator. The protons produce eight times as many π^+ as π^- in water target, and $\sim 3\%$ of the pions decay in flight ($\pi^+ \to \mu^+ \nu_\mu$), giving a ν_μ beam of up to about 180 MeV which goes into a tank 8.3 m long by 5.7 m in diameter. About 97% of the pions come to rest in a beam stop, and the resulting decay muons give a $\bar{\nu}_\mu$ beam via $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ of maximum energy 52.8 MeV into the 200 tons of mineral oil in the tank 30 m away. A small amount of scintillator is added to the oil so that the 1220 8-inch-diameter photomultiplier tubes (25% coverage) can detect both rings of Cherenkov and the slower scintillation light. Events from the two beams are separated because of their energy difference. Thus although $\nu_\mu \to \nu_e$ is being searched for via the decay-in-flight beam, and ν_e's are produced in the decay at rest, the latter are of too low an energy to produce electrons in the search window. The $\bar{\nu}_e$

background for the $\bar{\nu}_\mu$ beam comes mainly from the symmetrical decay chain starting with a π^-, but this is suppressed by the π^-/π^+ ratio, by 95% of the π^- being absorbed and not decaying, and by 88% of the μ^- being captured, giving a relative yield of only $(1/8)(0.05)(0.12) = 8 \times 10^{-4}$.

The more difficult analysis looking for $\nu_\mu \to \nu_e$ oscillations has not yet produced a useful outcome, but results from the 1993+1994 data for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations have already been published,[1] and the newer developments discussed here have been accepted for publication.[2] LSND detects $\bar{\nu}_e$ by $\bar{\nu}_e p \to e^+ n$, followed by a γ from $np \to d\gamma$ (2.2 MeV). Requiring an energy above 36 MeV eliminates most of the accidental background from $\nu_e\, {}^{12}\mathrm{C} \to e^- X$ (the detector not distinguishing between e^+ and e^-), while the upper energy requirement of 60 MeV allows for the $\bar{\nu}_\mu$ endpoint plus energy resolution.

The first step in searching for $\bar{\nu}_e$ interactions is to select electrons, and by far the biggest background is from cosmic rays. The hadronic component is eliminated by a 2 kg/cm^2 overburden, and a liquid scintillator veto shield enclosing all but the bottom of the detector rejects muons efficiently. To eliminate decay $e^\pm$ from muons, a veto is imposed on any event occurring too soon after even low-level activity in the detector or veto shield; a hardware cut of $\approx$ 7 muon lifetimes is extended in software to as much as $\approx$ 18 muon lifetimes. Electrons are distinguished from neutrons and residual muons by both Cherenkov and scintillation light. The Cherenkov ring and ratio of late (scintillation) to early (Cherenkov) light selects electrons, but further cosmic ray events are reduced by eliminating those having track centers closer than 35 cm to the phototube faces and removing events near the outer parts of the tank and headed inward. Any remaining cosmic ray background is well measured because about 14 times as much data are collected outside of the beam spills as inside.

Cosmic rays also serve a useful purpose, however. The very large sample of decay $e^\pm$ from stopped cosmic ray muons covers exactly the right energy range to provide an energy calibration, a means of measuring energy resolution, and a mechanism for tuning cuts in an unbiased manner. These decay electrons can be compared with the muons from which they were created and with cosmic ray neutrons, identified by their 2.2-MeV capture γ rays and initial small signal.

The second step in searching for $\bar{\nu}_e$ interactions is to require a correlated 2.2 MeV γ within a reconstructed distance, $\Delta r < 2.5$ m from the e^+, having a relative time, $\Delta t < 1$ ms, and a number of hit phototubes $21 \le N_\gamma \le 50$. To determine if such a γ is correlated with the e^+, a function R of Δr, Δt, and N_γ is defined to be the ratio of approximate likelihoods for it to be correlated (by comparing with distributions from the cosmic neutron sample) to its being accidental (using accidental photons seen with laser calibration events).

Using a strict criterion for reducing accidentals to a 0.6% rate (i.e., $R > 30$), and eliminating events with more than one γ, 22 events were observed for beam on and $36 \times 0.07 = 2.5$ events for beam off. The estimated beam-related background consisted of 1.72 ± 0.41 events with correlated neutrinos and 0.41 ± 0.06 without. The probability that the beam-on events are due entirely to a statistical fluctuation of the 4.6 ± 0.6 event expected total background is 4×10^{-8}. Figure 1(a) shows the energy distribution of all electrons before requiring a correlated photon (i.e., $R \geq 0$), and Fig. 1(b) shows that distribution when requiring $R > 30$.

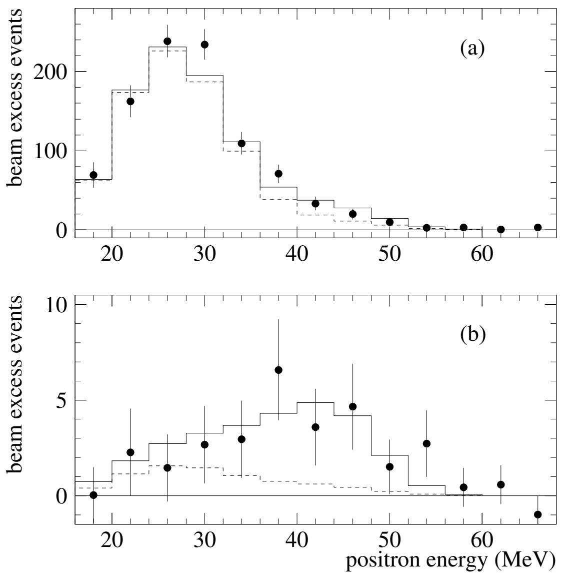

Figure 1: The energy distribution for positron events (a) without requiring a correlated γ ($R \geq 0$) and (b) with a tight requirement ($R > 30$) for a correlated γ. Shown in the figure are the beam-excess data, estimated neutrino background (dashed), and expected distribution for neutrino oscillations at large Δm^2 plus estimated neutrino background (solid).

While there are considerably more $R > 30$ events now than in the published report,[1] this is due more to changes in the analysis procedure than to additional data. Using the previous criteria,[1] there would be 13 events with an expected background of 4.3 ± 0.5, for which the probability of a fluctuation is

1×10^{-3}. Although Kolmogorov tests indicate no problem with the spatial distributions of the events, if the 55% of the tank volume is eliminated where the backgrounds are most intense, 6 events remain, whereas 1.7 ± 0.3 are expected, giving a fluctuation probability of 1×10^{-2}.

Statistically these events indicate the existence of neutrino oscillations. One may then ask whether these events have characteristics of oscillations. While a real proof of this would require changing the source-to-detector distance, Fig. 1 at least shows that the energy distribution of the $R > 30$ events is compatible with that expected from oscillations. Furthermore, Fig. 2, which shows the distribution in angle between the incident neutrino and the outgoing positron, is also oscillation like. The significance of this plot is enhanced by comparison with the quite different angular distributions provided by $\nu_e C \rightarrow e^- X$ and ν_e scattering.

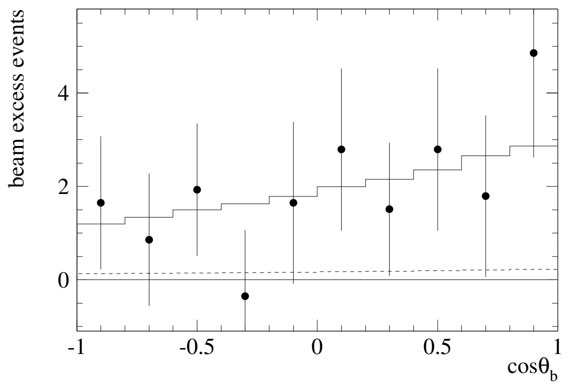

Figure 2: The $\cos\theta_b$ distribution for beam-excess data events with $36 < E_e < 60$ MeV and $R > 30$ and that expected for neutrino oscillations at large Δm^2 (solid). The dashed curve is the estimated neutrino background. θ_b is the e^+ angle with respect to the neutrino direction.

Assuming oscillations are being observed, one may use the simplification of a two-generation neutrino oscillation probability

$$P = (\sin^2 2\theta) \sin^2(1.27 \Delta m^2 L/E)$$

to describe the results. To determine favorable ranges of the mass-squared difference, Δm^2, as a function of the mixing angle, θ, between the two neutrino mass eigenstates, a fit is made to the observed event energy E, the neutron

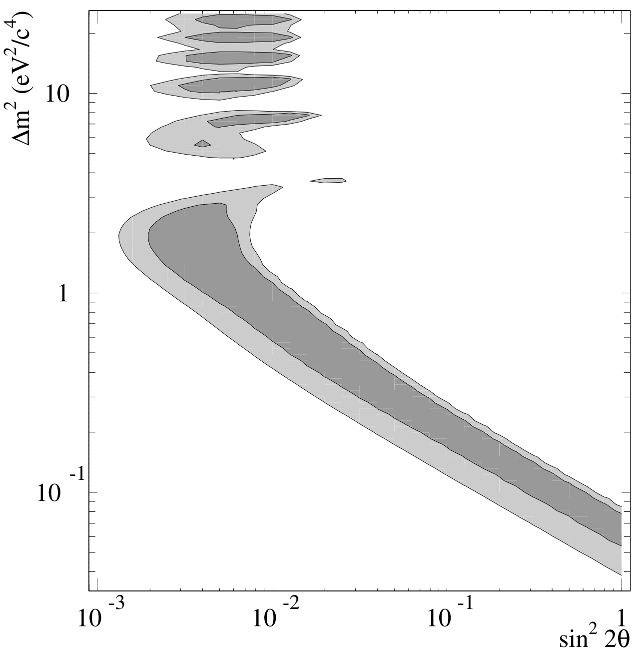

Figure 3: Mass-squared difference (Δm^2) vs. degree of mixing ($\sin^2 2\theta$) assuming a two-neutrino oscillation explanation of the LSND beam-excess data. Shown are regions of Δm^2 favored by a fit using the energy (from 20 to 60 MeV) and distance from the source of each event.

likelihood ratio R, the direction of the electron relative to the neutrino, and the distance of the event from the source L. In order to increase the range of L/E, values of E down to 20 MeV were used. Figure 3 shows favored regions of Δm^2, with contours at 2.3 and 4.5 log-likelihood units from the maximum. If this were a gaussian distribution, which it is not, the contours would correspond to 90% and 99% likelihood levels, but in addition they have been smeared to account for some systematic errors. Comparison to the KARMEN experiment,[3] which presents results in a similar way, shows no conflict, but if limits are plotted (as they are in Ref. 2) on this graph from E776 at BNL [4] and the Bugey reactor experiment,[5] then one might conclude that the only allowed Δm^2 region is 0.2–3 eV^2. If instead an 80% confidence level band is calculated to compare with the 90% confidence level limits of those experiments using, as they do, just numbers of events (i.e., not using the L/E information) and using only the 36–60 MeV region with its much lower background, then there is no conflict with other experiments above 0.2 eV^2.

If the same effect is seen in $\nu_\mu \to \nu_e$, then the case for neutrino mass is very strong, since the different detection mechanism ($\nu_e\,{}^{12}\mathrm{C} \to e^- X$) and the higher energy makes these two determinations essentially independent. The ν_e analysis is made more difficult because of the single signal of an e^- and because there is no calibration process like the $e^\pm$ from stopped muon decays for that high energy e^-. If $\nu_\mu \to \nu_e$ eventually shows an excess similar to that for $\bar{\nu}_\mu \to \bar{\nu}_e$, however, this would require that the observation is being made near an oscillation maximum, such as the 2, 6, and 10 eV^2 regions of Fig. 3. The maxima for the higher energy $\nu_\mu \to \nu_e$ are near 6, 18, ... eV^2, so a likely overlap is about 6 eV^2. These are rough numbers, and 6 could as well be 8, for example.

2 Possible Cosmological Consequences

While the LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ result could imply a range of $\Delta m^2 > 0.2$ eV, let us examine the consequences if $\nu_\mu \to \nu_e$ is also seen and Δm^2 is in the 6–8 eV^2 range. If, for definiteness, $\Delta m^2 = 6$ eV^2, either the ν_μ or ν_e is ≥ 2.4 eV, a value which would contribute $\gtrsim 10\%$ to the mass of the universe for critical density ($\Omega = 1$). This is a contribution to Ω equivalent to as much baryonic matter as nucleosynthesis permits and is more than an order of magnitude more than the baryonic matter observed so far. Models of the universe which utilize a density lower than critical (typically $\Omega \leq 0.3$) would then not have enough cold dark matter to produce sufficiently early structure formation, since the 2.4 eV neutrinos (hot dark matter) would wash out density fluctuations.[6] The only other models which come close to fitting the observed structure of the universe on all scales have $\Omega = 1$, so such a neutrino likely forces critical density, a requirement for an era of inflation and the only time-stable value for no cosmological constant.

One species of 2.4 eV neutrino would not provide a cold plus hot dark matter model which would fit structure information well. These models were created because the cold dark matter model (CDM), which was a fair approximation to the structure of the universe, when normalized to the COBE data, produced too much structure on small scales, since baryons readily clump around the cold dark matter. The first cold + hot dark matter models (CHDM) had $\sim 30\%$ neutrinos and fit structure on all scales very well[7] because the free streaming of the neutrinos reduced density fluctuations on small scales. Unfortunately, this damping of density perturbations also caused structure to form too late. Reducing the neutrino content to $\sim 20\%$ allowed early enough structure formation.[8] With all the mass in one neutrino species, this otherwise successful model (CνDM) overproduced clusters of galaxies. In other words, the CνDM model worked well at all distance scales except $\sim 10h^{-1}$Mpc, where h is the Hubble constant in units of 100 km$\cdot$s$^{-1}\cdot$Mpc^{-1}. If $h = 0.5$ and $\Omega = 1$, the mass of the neutrino required in the CνDM model is $94h^2\Omega F_\nu = 4.7$ eV, for a neutrino fraction of Ω, $F_\nu = 0.20$. If instead this mass is divided between two nearly degenerate neutrino species, the motivation for which will be discussed shortly, then this Cν^2DM model turned out to have a remarkable property.[6] While 4.7 eV in one neutrino species or two makes essentially no difference at very large or very small scales, at $\sim 10h^{-1}$Mpc the larger free-streaming length of the 2.4 eV neutrinos tends to wash out fluctuations and lowers the abundance of clusters. Thus the model (Cν^2DM) with two, 2.4 eV neutrinos fits structure information on all scales.

The Cν^2DM model works only if $\Omega = 1$ and if $h \lesssim 0.6$. The sets of determinations of h have been grouped near 0.5 and 0.7–0.8, with the latter being incompatible with $\Omega = 1$ and the age of the oldest stars. Recently there has been some convergence, with the upper values moving down and the lower values being more like 0.55, which for $F_\nu = 0.20$ would correspond to 2.8 eV neutrino mass or $\Delta m^2 = 8$ eV^2 for LSND. Should LSND determine Δm^2 and the masses needed for the Cν^2DM model be bourne out, then terrestrial experiments will have settled the long-disputed and all-important cosmological parameters Ω and h.

3 Possible Neutrino Mass Patterns

The reason the large-scale simulations for the Cν^2DM model were tried was a scenario[9] for neutrino masses designed to account for three hints of neutrino mass, one of which is the need for some hot dark matter. Of the other two, one is the deficit of ν_es from the sun observed by four experiments of three types. Of those three types, two have to be making incorrect observations in order for an astrophysical explanation of the deficit to work.[10] A solution in terms of neutrino oscillations requires that the mass-squared difference between the ν_e and whatever it turns into be no more than $\Delta m^2_{ei} \sim 10^{-5}$ eV^2. The second indication for neutrino mass results from evidence for a deficit of ν_μs relative to ν_es in atmospheric secondary cosmic rays. There are compatible results from three experiments[11] and further information from Kamiokande.[12] The latter includes accelerator confirmation of the ability to separate ν_e and ν_μ events, as well as an independent higher energy data set giving not only a ν_μ/ν_e ratio agreeing with the lower energy data, but also a zenith-angle (hence source-to-detector) dependence compatible with oscillations requiring $\Delta m^2_{\mu i} \sim 10^{-2}$ eV^2. The explanation of the observations in terms of the oscillation $\nu_\mu \to \nu_e$ is almost excluded by data from the Bugey[5] and Krasnoyarsk[13] reactor oscillation experiments. Also, in the higher energy Kamiokande data,[12] the muons display the zenith-angle dependence, whereas the electrons do not, as would be expected if the ν_μ were affected by oscillations, but the ν_e were not. Finally, the calculated ν_e and ν_μ fluxes—backed by measurements of μ fluxes—agree with the ν_e data but show a ν_μ deficit.[14] Thus $\nu_\mu \to \nu_\tau$ oscillations are favored as an explanation of this atmospheric ν_μ deficit.

If the solar ν_e and atmospheric ν_μ deficits and the need for some hot dark matter arise from the existence of neutrino mass, then there are basically two patterns of those masses.[9] Other schemes [15] are essentially rearrangements of those two patterns, and if LSND's evidence is correct, then one of those two patterns is ruled out. Invoking three-neutrino oscillations has difficulty in accommodating the three distinct mass differences needed,[16] and it appears likely that introducing a fourth (sterile) neutrino is necessary, as was done in the remaining one of the two mass patterns [9] just mentioned. The sterile neutrino, ν_s, which does not have the normal weak interactions and hence does not contribute to the width of the Z^0 boson, must still not exceed the limit on the number of relativistic species at the time of nucleosynthesis. This limitation, perhaps 3.5–4 — a subject of current controversy — can be avoided for the favored small mixing angle MSW or the vacuum oscillation solution for solar $\nu_e \to \nu_s$, since any other use of a light ν_s would bring it into equilibrium in the early universe. The ν_μ and ν_τ then provide the atmospheric ν_μ deficit and share the dark matter, making $m_{\nu_\mu} \approx m_{\nu_\tau} \approx 2.4$–$2.8$ eV. A combination of the SNO and SuperKamiokande experiments will be able to demonstrate $\nu_e \to \nu_s$ to check this mass pattern, which is consistent with the LSND oscillation result.

4 Supernovae Information

There is, however, a possible conflict between the $\bar{\nu}_\mu \to \bar{\nu}_e$ interpretation of the LSND data and information from supernovae. It is believed that much of heavy element production occurs in the outer neutrino-heated ejecta of Type II supernovae, where rapid interactions with the large number of neutrons can take place, the so-called r process. A limitation on the mixing of ν_μ and ν_e comes about because energetic ($\langle E \rangle \simeq 25$ MeV) ν_μ could convert via an MSW transition to ν_e inside the region where the r process is believed to occur. Such converted ν_e would have a much higher energy than the thermal ν_e ($\langle E \rangle = 11$ MeV), because the latter have charged current interactions with electrons and hence emerge from farther out in the supernova. Since the cross section for $\nu_e n \to e^- p$ rises rapidly with energy, the energetic ν_e would deplete the neutrons, stopping the r process. Calculations [17] of this effect limit $\sin^2 2\theta$ for $\nu_\mu \to \nu_e$ to $\lesssim 10^{-4}$ for $\Delta m^2_{\mu e} \gtrsim 2$ eV^2, apparently in conflict with the

LSND results.

Recently problems have been found with the r process itself which would require the region in which it takes place to be even more neutron rich, exacerbating the LSND conflict. A possible solution to these issues comes from inverting the neutrino mass hierarchy,[15] but a more likely solution is provided by the sterile neutrino, which produces two effects.[18] First, there is a zone outside the neutrinosphere but inside the $\nu_\mu \to \nu_e$ MSW region where the ν_μ interaction potential goes to zero, so a $\nu_\mu \to \nu_s$ transition can occur, reducing the dangerous high-energy $\nu_\mu s$. Second, because of the reduction in $\nu_\mu s$, the dominant process in the MSW region reverses, becoming $\nu_e \to \nu_\mu$, which reduces the ν_e flux into the r-process region, making it more neutron rich because of fewer $\nu_e n \to e^- p$ reactions.

The sterile neutrino may have two further uses in supernovae. While the r process goes on at rather late times (~ 10 s post bounce), another nucleosynthesis process goes on at early times (~ 1 s) and requires a proton-rich region. This p process could be aided by two regions where the ν_e interaction potential goes to zero, one inside the core and the other outside the neutrinosphere. The first converts $\nu_e \to \nu_s$, which then escape the dense core and reconvert at the second, producing high energy ν_es which produce protons via $\nu_e n \to e^- p$. These regions are effective only at these very early times and may provide the needed extra energy deposit to blow off the supernova mantle, since in present calculations the shock is stalled at an early time (0.15 s post bounce) close to the radius at which the $\nu_s \to \nu_e$ reconversion would occur at that time.

Thus there may be evidence for the sterile neutrino which had to be invoked in a rather ad hoc fashion to provide the small mass difference scale needed to explain the solar ν_e deficit, if the other active neutrinos are needed for the atmospheric ν_μ/ν_e ratio and both the hot dark matter component and the LSND result.

5 Conclusions

The LSND observation of 22 events of the type $\bar{\nu}_e p \to e^+ n$, $np \to d\gamma$, whereas 4.6 ± 0.6 were expected, is statistically strong evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations. If this is confirmed by a similar observation in the essentially independent $\nu_\mu \to \nu_e$ channel, then this will be the first laboratory violation of the Standard Model of particle physics and will likely provide evidence for a neutrino of 2–3 eV. Such a neutrino ensures that low-Ω models of the universe do not work, making it very likely that the universe has critical density ($\Omega = 1$). If the solar ν_e and atmospheric ν_μ deficits are also a result of neutrino mass, then the former can be explained by $\nu_e \to \nu_s$ (a sterile neutrino) and the latter by $\nu_\mu \to$

ν_τ. The ν_μ and ν_τ almost equally share the needed 5–6 eV for a cold + hot dark matter model of the universe which fits the structure on all scales, and besides requiring $\Omega = 1$, also needs a low value of the Hubble constant, eliminating problems of the universe age. Evidence for this pattern of neutrino masses may also be coming from the production of heavy elements in supernovae, since the ν_s may play a crucial role there. Thus the lightest elementary particle which can have mass and which has remarkably little interaction with other matter likely has profound effects in particle physics, astrophysics, and cosmology.

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